

The Fundamental Theorem of Calculus - Derivatives (Differentiating a Definite Integral)

Integrate the given integral using the Fundamental Theorem of Calculus for Area to find $F(x)$, and then differentiate $F(x)$ to find $F'(x)$.

$$1) F(x) = \int_1^x (t-3) dt$$

$$= \frac{1}{2}t^2 - 3t \Big|_1^x$$

$$F(x) - F(1)$$

$$= \left(\frac{1}{2}x^2 - 3x \right) - \left(\frac{1}{2} - 3 \right)$$

$$= \frac{1}{2}x^2 - 3x + 2.5$$

$$x-3$$

$$2) F(x) = \int_{x^2}^2 (t-3) dt$$

$$F(x) = \frac{1}{2}t^2 - 3t \Big|_{x^2}^2$$

$$F(2) - F(x^2)$$

$$F(x) = (-4) - \left(\frac{1}{2}x^4 - 3x^2 \right)$$

$$F(x) = -4 - \frac{1}{2}x^4 + 3x^2$$

$$F'(x) = -2x^3 + 6x = -2x(x^2 - 3)$$

$$3) F(x) = \int_1^{\sin x} (3u - 5) du$$

$$F(x) = \left. \frac{3}{2} u^2 - 5u \right|_1^{\sin(x)}$$

$$= \frac{3}{2} \sin^2(x) - 5 \sin(x) + 3.5$$

$$F'(x) = \sin(x) \cdot \cos(x) - 5 \cos(x)$$

$$= \cos(x) (3 \sin(x) - 5)$$

$$4) F(x) = \int_{\sin x}^3 (3u - 5) du$$

$$F'(x) = -\cos(x) (3 \sin(x) - 5)$$

$$F(x) = \left. \frac{3}{2} u^2 - 5u \right|_{\sin(x)}^3$$

$$= F(3) - F(\sin(x))$$

$$F(x) = \frac{27}{2} - 15 - \left(\frac{3}{2} \sin^2(x) - 5 \sin(x) \right)$$

$$F(x) = \frac{-3}{2} - \frac{3}{2} \sin^2(x) + 5 \sin(x)$$

$$F'(x) = -3 \sin(x) \cdot \cos(x) + 5 \cos(x)$$

$$= -\cos(x) (3 \sin(x) - 5)$$

Examine your answers to problems #1 through 4. Is there a short-cut method to finding $F'(x)$ for problems 5 and 6?

$$5) F(x) = \int_1^{x^3} (t^2 + t) dt$$

$$6) F(x) = \int_{\tan x}^5 (t^2 + t) dt$$

$$F'(x) = 3x^2(x^6 + x^3) - 0(1^2 + 1)$$

$$F'(x) = 3x^2(x^6 + x^3)$$

$$F'(x) = 0(5^2 + 5) - \sec^2(x)$$

$$F'(x) = -\sec^2(x)(\tan^2(x) + 1)$$

The Fundamental Theorem of Calculus for Derivatives
(Differentiating a Definite Integral)

$$D_x \left[\int_{h(x)}^{g(x)} f(t) dt \right] = f(g(x)) \cdot g'(x) - f(h(x)) \cdot h'(x)$$

Find $F'(x)$ using the Fundamental Theorem of Calculus for Derivatives.

1) $F(x) = \int_1^x (t^2 + 2t - 5) dt$

$$\begin{aligned} F'(x) &= 1 \cdot (x^2 + 2x - 5) \\ &= x^2 + 2x - 5 \end{aligned}$$

2) $F(x) = \int_{x^3}^4 \sin^3 t \cos t dt$

$$F'(x) = -3x^2 \sin^3(x^3) \cos(x^3)$$

$$3) F(x) = \int_1^{x^2} (5t^4 + 2t^2 - 4t) dt$$

$$F'(x) = 2x(5x^8 + 2x^4 - 4x^2)$$

$$4) F(x) = \int_8^{x^2} \sqrt{s^3 + 5} ds$$

$$F'(x) = 2x \sqrt{x^6 + 5}$$

$$5) F(x) = \int_1^{2x^4 - 3x} (5u - 7) du$$

$$\begin{aligned} F'(x) &= (8x^3 - 3)(5(2x^4 - 3x) - 7) \\ &= (8x^3 - 3)(10x^4 - 15x - 7) \end{aligned}$$

$$6) F(x) = \int_{3x-4}^9 (u \sec u) du$$

$$F'(x) = -3((3x-4) \cdot \sec(3x-4) + \tan(3x-4))$$

$$7) F(x) = \int_1^{\tan x} \frac{1}{1+t^2} dt$$

$$F'(x) = \sec^2(x) \left(\frac{1}{1+\tan^2(x)} \right)$$

$$8) F(x) = \int_{\cos x}^2 \tan u \, du$$

$$F'(x) = \sin(x) \cdot \tan(\cos(x))$$

$$9) F(x) = \int_1^{\sin x^3} (t^2 + t) \, dt$$

$$(x^3) \cdot 3x^2 (\sin^2(x^3) + \sin(x^3)) \quad F'(x) = \sec(x) \tan(x) (\sec^3(x) - 3 \sec^2(x) \sec^2(x) (\tan^3(x) - 3 \tan^2(x)))$$

$$10) F(x) = \int_{\tan x}^{\sec x} (t^3 - 3t^2) \, dt$$

Calculus I

Wkst - Fundamental Th. of Calculus - Derivative

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$$D_x \left[\int_{h(x)}^{g(x)} f(t) dt \right] = f(g(x)) \cdot g'(x) - f(h(x)) \cdot h'(x)$$

Find $F'(x)$ using the Fundamental Theorem of Calculus - Derivatives.

$$1) F(x) = \int_1^x (t^3 + 2t - 4) dt$$

$$F'(x) = x^3 + 2x - 4$$

$$3) F(x) = \int_2^{x^2+1} \sqrt{2 + \sin u} du$$

$$F'(x) = 2x \sqrt{2 + \sin(x^2+1)}$$

$$2) F(x) = \int_4^{\sin x} (u^2 - 2) du$$

$$F'(x) = \cos(x) (\sin^2(x) - 2)$$

$$4) F(x) = \int_{x^2}^2 \cos(4s) ds$$

$$F'(x) = -2x \cos(4x^2)$$

$$5) F(x) = \int_{2x^2+x}^1 \sqrt{1+s^4} ds$$

$$F'(x) = -(4x+1) \sqrt{1+(2x^2+x)^4}$$

$$6) F(x) = \int_{\sin x}^{\cos x} u^2 du$$

$$F'(x) = -\sin(x) \cdot \cos^2(x) - \cos(x) \cdot \sin^2(x)$$

